

Final Exam MTH 221 , Summer 2022

Ayman Badawi

Score = $\frac{}{62}$

QUESTION 1. (20 points)

(i) Let $T : R^{2\times 2} \to R^{2\times 2}$ be an R-homomorphism, i.e., linear transformation, such that $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} 3a-b+2d & 4b+7d \\ 5c+d & 6d \end{bmatrix}$. Then the eigenvalues of T

(a) 3, 7, 5, 6 (b) 3, 4, 5, 6 (c) 3, 4, 1, 6 (d) 3, 4, 7, 6

(ii) Given A is a 2×2 matrix with eigenvalues $1, 2$, such that $E_1 = span\{(3,0)\}$ and $E_2 = span\{(0,4)\}$ Then A^3 =

(a) $\begin{bmatrix} 27 & 0 \\ 0 & 64 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 64 \\ 0 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 27 \\ 0 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ such that $|A| = 0$. Let D be the solution set of the system of linear equations $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2a_1+a_2 \\ -2b_1+b_2 \\ -2c_1+c_2 \end{bmatrix}$. Then

(a) $D = span\{(-2,1,0)\}$ (b) $D = \{(-2,0,1)\}$ (c) D is infinite and $(-2,1,0) \in D$. (d) $D = \{(-2,1,0)\}$

(iv) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ such that $|A| = -3$. Given $B = \begin{bmatrix} -2a_1 & a_2 & a_3 \\ -2c_1 & c_2 & c_3 \\ -2b_1 & b_2 & b_3 \end{bmatrix}$. Then $|B| =$

(a) -6 (b) 6 (c) 24 (d) -24

(v) Let A be a 3×3 matrix such that $C_A(\alpha) = (\alpha-1)(\alpha-b)(\alpha-c)$, where $b, c \in R$, $Trace(A) = 1$ and $|A| = -9$. Then $|A^2 + I_3|$ is

(a)82 (b) 10 (c) 100 (d) 200

(vi) Let $T : P_3 \to P_3$ be an R-homomorphism (linear transformation), such that $T(ax^2+bx+c) = (3a-b-c)x^2 + 3bx + 3c$. Then T has exactly one eigenvalue, say a, then $E_a =$

(a) $span\{x^2,\ x+1\}$ (b) $span\{x-1\}$ (c) $spam\{x^2,\ x-1\}$ (d) $Span\{-x-1\}$

(vii) Assume that the normal dot product is defined on R^4. Given $\{Q,\ F,\ (1,0,0,2)\}$ is an orthogonal basis for a subspace W of R^4, for some points Q, F in R^4. Given $(4,\ 23,\ 51,\ 13) \in W$. Then $(4,23,51,13) = c_1Q + c_2F + c_3(1,0,0,2)$. Then $c_3 =$

(a) 30 (b) 6 (c) 5 (d) 10

(viii) Given $B = \{(-6,-1),\ (7,1)\}$ is a basis for R^2. Then $[(13,\ 2)]_B =$

(a) $(-1,1)$ (b) (15, -103) (c) (-80, 93) (d) (27, -25)

(ix) Let B be a basis for R^3 and $C = \{(2,2),\ (1,2)\}$ is a basis for R^2. Let $T : R^3 \to R^2$ be a linear transformation such the coordinate matrix presentation of T with respect to B and C is $[T]_{B,C} = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -1 & 1 \end{bmatrix}$. Then $T(2,0,1) =$

(a) (4, -1) (b)(5, -1) (c) (9, 8) (d) (7, 6)

(x) consider the "mimic dot product" on $R^{2\times 2}$, i.e., for every $A, B \in R^{2\times 2}$, $< A, B >= Trace(B^T A)$. Then the following matrices are orthogonal

(a) $A = \begin{bmatrix} 1 & 4 \\ -2 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ 6 & -2 \end{bmatrix}$ (b) $A = \begin{bmatrix} -3 & -2 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 3 \\ -15 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} -3 & -2 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 3 \\ 15 & -1 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 \\ -1 & -1 \end{bmatrix}$

QUESTION 2. (i) (4 points) consider the "mimic dot product" on $R^{2\times 2}$, i.e., for every $A, B \in R^{2\times 2}$, $< A, B >= Trace(B^T A)$. Let $W = span\{\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ -4 & 0 \end{bmatrix}\}$. Use Gram-Schmidt algorithm and find an orthogonal basis for W.

(ii) (4 points) consider the "integral inner product" on P_3, i.e., for every $f(x), k(x) \in P_3$, $< f(x), k(x) >= \int_0^1 f(x)k(x)\, dx$. Find the distance between $f(x) = 2x^2 + x + 1$ and $k(x) = x^2 + 1$.

QUESTION 3. Let $B = \{(1, 0, -1), (0, 1, -1), (1, 0, 0)\}$ be a basis for R^3 and $C = \{(1, 0), (0, 1)\}$ be a basis for R^2. Given $T : R^3 \to R^2$ is an R-homomorphism (i.e., Linear Transformation) such that $T(1, 0, -1) = (1, 0)$, $T(0, 1, -1) = (1, 0)$, and $T(1, 0, 0) = (1, 0)$.

(i) (6 points) Find the coordinate matrix presentation of T with respect to B and C, $[T]_{B,C}$

(ii) (3 points) Find $T(2, 1, 1)$

(iii) (5 points) Find $Z(T) = Ker(T) = Null(T)$

QUESTION 4. (i) (4 points) Assume the normal dot product on R^4. Let $W = span\{(1,1,1,1),(0,1,0,0)\}$. Find a basis for $W^{\perp}$

(ii) (4 points) Given A is a 3×5 matrix such that $A \xrightarrow{2R_2} B \xrightarrow{R_1 \leftrightarrow R_3} C \xrightarrow{2R_1+R_3 \rightarrow R_3} D$. Find elementary matrices E_1, E_2, E_3 such that $E_1E_2E_3A = D$

(iii) (4 points) Given $A = (2,4), B = (-1,6)$ and $C = (3,7)$. Find the area of the triangle ABC.

QUESTION 5. (8 points) Let $W = span\{(1,0,0,0,0),\ (0,1,0,0,0),\ (0,0,1,0,0)\}$ and $D = span\{(1,1,0,0,0),(0,0,0,0,1),(0,0,0,1,1)\}$

(i) Find a basis for $W \cap D$.

(ii) Find a basis for $W + D$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com